Instructions: This quiz has 77 questions. The use of calculators is forbidden. Click on the box with the right answer. To initialise the quiz you must click on "BEGIN QUIZ." When you finish the quiz you click on "END QUIZ" in order to see your score.

Begin Quiz Answer each of the following.

1. Find $\lim_{x\to a} x^2 - ax + b$, where a, b are real constants.

☐ a ☐ ab ☐ b ☐ $a^2 + b$

2. Find $\lim_{x\to a} \dfrac{x^2 - a^2}{x - a}$, where a is a real constant.

☐ a ☐ $2a$ ☐ 0 ☐ a^2

3. Let $f(x) = \begin{cases} 1 & \text{if } x \neq 1 \\ -1 & \text{if } x = 1 \end{cases}$. Which of the following assertions is true?

☐ $\lim_{x\to 1} f(x) = 1$ and f is continuous at 1

☐ $\lim_{x\to 1} f(x) = -1$ and f is continuous at 1

☐ $\lim_{x\to 1} f(x) = 1$ and f is discontinuous at 1

☐ $\lim_{x\to 1} f(x) = -1$ and f is discontinuous at 1

4. If $f(x) = ax^2 - a^2x$, where a is a real constant, then $f'(a) =$

☐ 0 ☐ $2a$ ☐ a ☐ a^2

5. If $y = \dfrac{b}{x - a}$, where a, b are real constants, then $\dfrac{\mathrm{d}y}{\mathrm{d}x} =$

☐ $-\dfrac{b}{(x-a)^2}$ ☐ $\dfrac{b}{(x-a)^2}$ ☐ $\dfrac{b}{x(x-a)}$ ☐ $b(x-a)$

6. f is a function such that $f(0) = 0$, $xf'(x) > 0$ for $x \neq 0$, and $xf''(x) > 0$ for $x \neq 0$. Which of the following could be the graph of f?

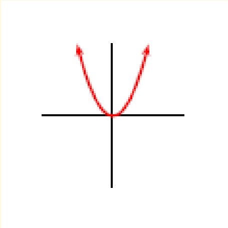

Figure 1: I

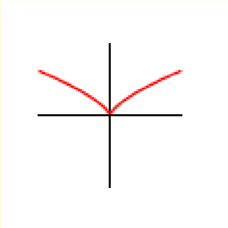

Figure 2: II

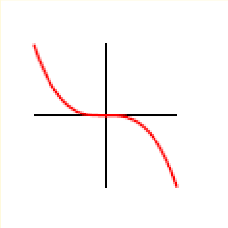

Figure 3: III

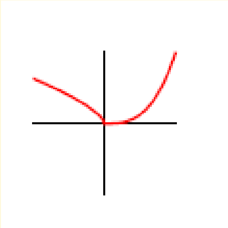

Figure 4: IV

☐ I ☐ II ☐ III ☐ IV

7. At $x = -2$, the tangent line to the graph of $y = x^3 - 2x^2 + 1$ has slope

☐ -20 ☐ -15 ☐ 20 ☐ 4

8. $\int 3x^5 \mathrm{d}x =$

☐ $\frac{x^6}{2} + C$ ☐ $\frac{x^6}{3} + C$ ☐ $\frac{x^5}{2} + C$ ☐ $15x^2 + C$

9. By the Mean Value Theorem, between $(1, 0)$ and $(2, 1)$, the function $f(x) = x^3 - 2x^2 + 1$ has at least one point at which its derivative has the value

☐ -1 ☐ 0 ☐ 1 ☐ 2

10. If $a \neq 0$ is a real constant, then $\int \csc^2 ax \mathrm{d}x =$

☐ $\frac{\cot ax}{a} + C$ ☐ $-\frac{\cot ax}{a} + C$ ☐ $\frac{(\csc ax)^3}{a} + C$ ☐ $\csc^3 ax + C$

11. If $a \neq 0$ is a real constant, and $f(x) = (\sin ax)(\cos ax)$, then $f'(0) =$

☐ 0 ☐ a ☐ $2a$ ☐ a^2

12. If a, b, c are non-zero real constants, then $\lim_{x\to+\infty} \frac{ax^2 + bx + c}{a^2x^2 + b^2x + c} =$

☐ ab ☐ a ☐ $2a$ ☐ $\frac{1}{a}$

13. The slope of the line tangent to the curve $x + 4y - 2y^3 - 3 = 0$ at the point $(1, 1)$ is

☐ $-\frac{1}{3}$ ☐ $\frac{1}{3}$ ☐ $\frac{1}{2}$ ☐ $-\frac{1}{2}$

14. The absolute maximum value of the function $f(x) = x^3 - 2x + 4$ on the interval $[-2; 2]$ is

☐ $4 - \frac{4}{9}\sqrt{6}$ ☐ $\frac{4}{9}\sqrt{6} + 4$ ☐ 8 ☐ 10

15. If $f(0) = a$ and $f(1) = b$, where a, b are real constants, then $\int_0^1 f'(x)\mathrm{d}x =$

☐ $a - b$ ☐ $b - a$ ☐ $a + b$ ☐ impossible to determine

16. Compute the area between the curves $y = 2 - 2x - x^2$ and $y = 1 - 2x$.

☐ 1 ☐ $\frac{2}{3}$ ☐ $\frac{4}{3}$ ☐ $\frac{1}{3}$

17. Figure 5 consists of four semicircles, two of radius 1 and two of radius $\frac{1}{2}$. If $g(x) = \int_0^x f(t)\mathrm{d}t$, where is $g(x) \geq 0$?

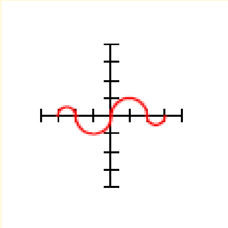

Figure 5: Problem 17.

- ☐ $[-3; 3]$
- ☐ $[-3; -2] \cup [0; 2]$ only
- ☐ $[0; 3]$ only
- ☐ $[-3; -2] \cup [0; 3]$ only

18. The number of unfilled pharmacist positions in year x is approximately

$$P(x) = -583x^3 + 2068x^2 + 323x + 2670, \qquad 0 \leq x \leq 3.$$

If $x = 0$ corresponds to year 1998, in what year was the shortage of pharmacists most severe?

☐ 1998 ☐ 1999 ☐ 2000 ☐ 2001

19. When a stone is dropped into a pool, a circular wave moves out from the point of impact at the rate of $3a$ inches per second ($a > 0$ a real constant). How fast is the area enclosed by the wave increasing when the radius of the wave is a inches?

- ☐ $2\pi a^2$ square inches per second
- ☐ $3\pi a^2$ square inches per second
- ☐ $4\pi a^2$ square inches per second
- ☐ $6\pi a^2$ square inches per second

20. $\displaystyle\int_1^9 \frac{\mathrm{d}x}{(2+\sqrt{x})^2\sqrt{x}} =$

☐ $\dfrac{196}{10125}$ ☐ $\dfrac{4}{15}$ ☐ $2\ln\frac{5}{3}$ ☐ $\dfrac{16}{225}$

21. If Newton's method is used to solve $x^3 - 3x^2 + 4x - 1 = 0$ with an initial guess of $x = 0$, then the second approximation is closest to

☐ 0.318 ☐ 0.314 ☐ 0.25 ☐ 0.184

22. For which value of the real parameter a does the polynomial $x^3 + 2x + a$ have a root in the interval $[-1; 0]$?

☐ $a = -1$ ☐ $a = 0$ ☐ $a = 10$ ☐ $a = 20$

23. If a is a real constant, then $\lim_{x\to 0} \dfrac{\sin ax - \tan a^2 x}{x} =$

☐ $2a$ ☐ $-a$ ☐ $a - a^2$ ☐ a

24. $\lim_{h\to 0} \dfrac{\sqrt[4]{16+h} - 2}{h} =$

☐ $\dfrac{1}{16}$ ☐ $\dfrac{1}{32}$ ☐ $\dfrac{1}{64}$ ☐ $\dfrac{1}{4}$

25. Consider the three functions

$$a(x) = \sqrt{\frac{2-x}{x+1}}; \qquad b(x) = \sqrt{2-x} + \sqrt{x+1}; \qquad c(x) = \sqrt{\frac{x+1}{2-x}}$$

and the three sets of real numbers

$$I : [-1;2] \qquad II :]-1;2] \qquad III : [-1;2[.$$

Match each function with its domain of definition.

☐ (I,a), (II,b), (III,c)
☐ (I,b), (II,c), (III,a)
☐ (I,b), (II,a), (III,c)
☐ (I,c), (II,b), (III,a)

26. $f : A \to B$ is a function, where $f(x) = \dfrac{\sqrt{1-x}}{(x-2)(x+1)}$ and A and B are subsets of the real numbers. Which of the following values of x could never belong to A?

☐ $x = 0$ ☐ $x = 1$ ☐ $x = -2$ ☐ $x = 3$

27. $f : \mathbb{R} \to \mathbb{R}$ is a function such that $f(2x-3) = x^3$. Find $f(1)$.

☐ 1 ☐ -1 ☐ 8 ☐ $\dfrac{1}{8}$

28. If $f : \mathbb{R} \to \mathbb{R}$ is a function such that $f(1-2x) = 4x^2$, then $(f \circ f)(x) =$

☐ $4x^2(1-2x)^2$ ☐ $x^2(1-x)^2$ ☐ $x^2(2+x)^2$ ☐ $x^2(2-x)^2$

29. If $f(x) = \dfrac{1}{\sqrt{1-x^2}}$ is a real number, then $x \in$

☐ $]+1;+\infty[$ ☐ $]-\infty;+1[$ ☐ $[-1;+1]$ ☐ $]-1;+1[$

30. Given that

$$f(x) = \begin{cases} 6 + x & x \le -2 \\ 3x^2 + xa & x > -2 \end{cases}$$

is continuous, find a.

☐ -2 ☐ -4 ☐ 8 ☐ 4

31. $\lim_{x\to 0} \dfrac{\sqrt{1+2x} - \sqrt{1-2x}}{x} =$

☐ 0 ☐ $\frac{1}{2}$ ☐ 1 ☐ 2

32. $\lim_{x\to 0} \dfrac{\tan 6x}{\sin \dfrac{x}{2}} =$

☐ 3 ☐ $\frac{1}{3}$ ☐ 12 ☐ $\dfrac{1}{12}$

33. If $\sin \frac{1}{x^2} \le f(x) \le \cos \frac{1}{x^2}$ for $x \ne 0$, then $\lim_{x\to 0} x^4 f(x) =$

☐ $+\infty$ ☐ -1 ☐ 1 ☐ 0

34. Let f be a twice differentiable function with $f'(0) = 0$. Then $\lim_{x\to 0} \dfrac{f'(x)}{x} =$

☐ 0 ☐ $f'(x)$ ☐ $f''(0)$ ☐ $f''(x)$

35. Let f be a differentiable function. Determine

$$\lim_{x\to+\infty} x\left(f\left(\frac{1}{x}\right) - f(0)\right).$$

☐ $+\infty$ ☐ $f'(0)$ ☐ 0 ☐ $f(1) - f(0)$

36. Let f be a strictly increasing continuous function with $f(3) = 5$. Find

$$\lim_{x\to 3} \frac{((f(x))^2 - 25)}{f(x) - 5}.$$

☐ 0 ☐ 5 ☐ 10 ☐ 15

37. If $f(x) = e^2$ then $f'(x) =$

☐ e^2 ☐ $2e^2$ ☐ 0 ☐ 1

38. If $f(x) = 4^x$ then $f'(x) =$

☐ 4^x ☐ $x4^{x-1}$ ☐ $\frac{4^x}{\ln 4}$ ☐ $(\ln 4)4^x$

39. If $f(x) = \log_4 x$ then $f'(x) =$

☐ $\frac{\ln 4}{x}$ ☐ $\frac{1}{\log_4 x}$ ☐ $\frac{1}{x \ln 4}$ ☐ $(\ln 4)4^x$

40. Let g be a differentiable function such that

$$g(x + h) - g(x) = 2xh + h^2 - 2h$$

and $g(0) = 2$. Find $g(x)$.

☐ $x^2 + 2x + 2$ ☐ $x^2 - x + 2$ ☐ $x^2 - 2x + 2$ ☐ $x^2 + 2$

41. Let f be a differentiable function such that

$$f(x + h) - f(x) = e^{x+h} - h - e^x$$

and $f(0) = 3$. Find $f(x)$.

☐ $xe^x - e^x + 3$ ☐ $e^x + 2$ ☐ $4 - e^x$ ☐ $e^x - x + 2$

42. f is differentiable at $x = 2$ and $g(x) = (x - 2)^2 f(x)$. Then $g'(2) =$

☐ < 0 ☐ > 0 ☐ $= 0$ ☐ $= f'(2)$

43. If $g(x) = \dfrac{x}{f(x)} + 1$, $f(-1) = 2$ and $f'(-1) = -1$, find $g'(-1)$.

☐ $\frac{3}{4}$ ☐ $\frac{1}{4}$ ☐ $-\frac{1}{2}$ ☐ $\frac{1}{2}$

44. If $g(x) = f(x^e + ex)$, $f(e+1) = 0$, and $f'(e+1) = 2$, find $g'(1)$.

☐ $4e$ ☐ $2e$ ☐ e ☐ 0

45. $f(x) = x^{\sin x}$. Find $f'\left(\frac{\pi}{2}\right)$.

☐ $\frac{2}{\pi} + \frac{\pi}{2}$ ☐ 1 ☐ $\frac{\pi}{2}$ ☐ $\frac{2}{\pi}$

46. T is the tangent to the graph of $x^2 + y^2 - 8x - 9 = 0$ at the point on the graph in the fourth quadrant where $x = 1$. At what point does T intersect the graph of $x + 4 = 0$?

☐ $(-4, 4)$ ☐ $(1, -4)$ ☐ $(-4, -\frac{1}{4})$ ☐ $(-4, -\frac{25}{4})$

47. The point $(9, t)$ lies on the line tangent to $x^2 + xy + y^3 + 1 = 0$ at $(1, -1)$. Find t.

☐ -1 ☐ -3 ☐ -5 ☐ $-\frac{9}{4}$

48. Find the absolute maximum of $g(x) = \sqrt{24 + 2x - x^2}$ on the interval $0 \le x \le 5$.

☐ 0 ☐ 1 ☐ 5 ☐ $3\sqrt{3}$

49. The derivative of a function f is $f'(x) = x^2(x-1)^4(x-3)^3$. How many local maxima does f have?

☐ 0 ☐ 1 ☐ 2 ☐ 3

50. The inflexion point for $f(x) = \dfrac{\ln x}{x}$ occurs at $x =$

☐ e^{-1} ☐ $\sqrt{e}$ ☐ e ☐ $e^{3/2}$

51. The absolute minimum of $f(x) = \sin 2x - 2\sin x$ on the interval $[0; 2\pi]$ occurs at $x =$

☐ 0 ☐ $\frac{\pi}{2}$ ☐ $\frac{2\pi}{3}$ ☐ $\frac{3\pi}{2}$

52. $\displaystyle\int_{-e}^{-e^{-1}} \frac{dx}{x} =$

☐ -2 ☐ 0 ☐ 1 ☐ 2

53. $\displaystyle\int_{0}^{\pi} \left(6x^2 \sin x^3 - \frac{2}{\pi}\right) \, dx =$

☐ $2\cos(\pi^3)$ ☐ $-2\sin(\pi^3)$ ☐ $-\cos(\pi^3)$ ☐ $-2\cos(\pi^3)$

54. $\displaystyle\int_{1/2}^{2} \frac{dx}{x^2} =$

☐ $-\frac{3}{2}$ ☐ 2 ☐ $\frac{1}{2}$ ☐ $\dfrac{3}{2}$

55. $\displaystyle\int_{0}^{\pi/2} \frac{\sin x dx}{1+\cos x} =$

☐ $\ln \frac{\pi}{2}$ ☐ 2 ☐ $\ln \frac{1}{2}$ ☐ $\ln 2$

56. $\displaystyle\int_{0}^{7} \sqrt[3]{x+1} \, dx =$

☐ $\dfrac{41}{4}$ ☐ $\dfrac{37}{4}$ ☐ $\dfrac{45}{4}$ ☐ $\dfrac{47}{4}$

57. $\displaystyle\int_{e-1}^{e+1} \frac{dx}{x+e} =$

☐ $\ln \dfrac{e+2}{e-2}$ ☐ $\ln \dfrac{2e+1}{2e-1}$ ☐ $\ln(4e^2-1)$ ☐ $\ln \dfrac{e+1}{e-1}$

58. $\displaystyle\int_{1}^{2} \frac{x^3 dx}{1+x^4} =$

☐ $\dfrac{15}{8}$ ☐ $\dfrac{1}{4} \ln \dfrac{17}{2}$ ☐ $\ln \dfrac{17}{2}$ ☐ $\dfrac{15}{4} + \ln 2$

59. $\displaystyle\int_1^2 \frac{(1+x^4)\mathrm{d}x}{x^3} =$

☐ $\frac{15}{8}$ ☐ $\frac{1}{4}\ln\frac{17}{2}$ ☐ $\ln\frac{17}{2}$ ☐ $\frac{15}{4}+\ln 2$

60. $\displaystyle\int_0^{\pi/4} \frac{(\cos x - \sin x)\mathrm{d}x}{\sin x + \cos x} =$

☐ $\frac{1}{2}\ln 2$ ☐ $\frac{1}{4}\ln 2$ ☐ 0 ☐ $\ln 2$

61. $\displaystyle\int_{-1}^2 |x|\mathrm{d}x =$

☐ $\frac{3}{2}$ ☐ $\frac{5}{2}$ ☐ 1 ☐ 3

62. Let $\llbracket x \rrbracket$ denote the greatest integer less than or equal to x. Then $\displaystyle\int_1^2 \llbracket x^2 \rrbracket \mathrm{d}x =$

☐ $\frac{7}{3}$ ☐ 5 ☐ $5-\sqrt{2}-\sqrt{3}$ ☐ 6

63. f is increasing with $f(0)=0$. If f has a continuous derivative and $\displaystyle\int_0^1 f(x)f'(x)\mathrm{d}x = 3$, find $f(1)$.

☐ $\sqrt{2}$ ☐ 6 ☐ $\sqrt{3}$ ☐ $\sqrt{6}$

64. Find the area of the region in the plane bounded by the curves $y^2 = 4x$ and $2x - y = 4$.

☐ 0 ☐ $\frac{4}{3}$ ☐ $\frac{2}{3}$ ☐ $\frac{1}{3}$

65. Calculate the area of the closed region in the xy-plane bounded by $y = x - 5$ and $y^2 = 2x + 5$.

☐ 8 ☐ $\frac{128}{3}$ ☐ $\frac{122}{3}$ ☐ $\frac{98}{3}$

66. Find $\int_0^2 \max(x, x^2)$, where $\max(f, g)$ denotes the maximum value of f and g.

- [ ] $\dfrac{8}{3}$
- [ ] $\frac{17}{6}$
- [ ] 2
- [ ] $\frac{14}{3}$

67. For which value of c is

$$f(x) = \begin{cases} 3x^2 + 2, & x \geq -1 \\ -cx + 5, & x < -1 \end{cases}$$

continuous?

- [ ] 0
- [ ] -5
- [ ] 5
- [ ] 10

68. $\lim\limits_{x\to+\infty} \sqrt[3]{\dfrac{7 - x^2}{x(x+1)}} =$

- [ ] 0
- [ ] 1
- [ ] $\sqrt[3]{6}$
- [ ] -1

69. $\lim\limits_{x\to 0^-} \dfrac{x}{|x|} =$

- [ ] $-\infty$
- [ ] -1
- [ ] $+\infty$
- [ ] 1

70. $\lim\limits_{x\to 0^+} (1 - x)^{2/x}$

- [ ] e^{-2}
- [ ] $e^{-1/2}$
- [ ] e^{2}
- [ ] $e^{1/2}$

71. $\lim\limits_{x\to 1^-} \dfrac{1}{x - 1}$

- [ ] $+\infty$
- [ ] 0
- [ ] $-\infty$
- [ ] 1

72. $\frac{d}{dx}\log_3(x^2+1) =$

☐ $\frac{2x}{\log_3(x^2+1)}$

☐ 1

☐ $\frac{2x}{(x^2+1)\ln 3}$

☐ $\frac{1}{x^2+1}$

73. The slope of the line tangent to the curve $y^3 - 3x^3 + x^2y^2 = 9$ at $(1,\, 2)$ is

☐ $\frac{1}{16}$ ☐ $\frac{1}{12}$ ☐ $\frac{1}{4}$ ☐ $\frac{5}{16}$

74. F and G are differentiable functions on the interval $[-1;+1]$ with $F(0) = G(0) = 0$. If F is increasing and G is decreasing then $\lim_{x\to 0}\frac{F(x)}{G(x)}$

☐ $= 0$ ☐ < 0 ☐ > 0 ☐ Does not exist

75. Let f be a continuous real-valued function that only assumes rational values. If $f(1) = -1$, then $f(-1) =?$

☐ 1 ☐ -1 ☐ 0 ☐ Impossible to determine

76. If $f(x) = x^3e^{-x}$, and $f^{(n)}(x)$ denotes the n-th order derivative of f, what is $f^{(100)}(0)$?

☐ 1 ☐ -970200 ☐ 0 ☐ 970200

77. Find the volume of the solid when the region bounded by the x-axis, the y-axis, and the curve $y = x(1-x)$ is rotated about the x-axis.

☐ $\frac{\pi}{30}$ ☐ $\frac{\pi}{15}$ ☐ $\frac{\pi}{6}$ ☐ $\frac{\pi}{60}$

End Quiz

Score: | Correct

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