Math 271 **Final (Version 4)**

Name........................J Number..................Score..................

Show all work

(1) Find an equation of the plane that contains the line $\begin{cases} x = t \\ y = 2t + 3 \\ z = -t + 8 \end{cases}$, and parallel to the line $\frac{x-1}{2} = y = z$. (10)

(2) Find the limits. (10)
   (a). $\lim_{(x,y)\to(0,0)} \frac{\sqrt{x^2+4y^2+4}-2}{x^2+y^2}$;
   (b). $\lim_{(x,y)\to(0,0)} \sqrt{x+y}\ln(x^2+y^2)$.

(3) Find the partial derivatives of z with respect to x, and y respectively: (10)
   (a). $z = e^s \cos t$, where $s = x + y, t = \sqrt{x^2 + y^2}$;
   (b). $\ln(x + 2yz) = 2 - xyz^2$.

(4) The temperature T at a point (x, y) on a semi-circular plate is given by $T(x, y) = 3x^2y - y^3 + 273$. (10)
   (a). Find the temperature at $(1, 2)$;
   (b). Find the rate of change of temperature at $(1, 2)$ in the direction of $\overrightarrow{a} = \overrightarrow{i} - 2\overrightarrow{j}$;
   (c). Find a unit vector in the direction in which the temperature increases most rapidly at $(1, 2)$ and find this maximum rate of change.

(5) Consider the surface $z = x^2 + \frac{y^3}{3} - 2xy - y$.(10)
   (a). Find all the critical points on this surface;
   (b). At each of the critical points determine if a relative maximum, relative minimum or saddle point occurs.

(6) Find the mass and the center of mass $(\overline{x}, \overline{y})$ of the region D bounded by the x-axis and $y = -x^2 + 16$ with the density function $\rho(x, y) = 2x$. (10)

(7) Evaluate the integral by changing it to the spherical coordinates:
$\int_0^2 \int_0^{\sqrt{4-x^2}} \int_{\sqrt{3(x^2+y^2)}}^{\sqrt{16-x^2-y^2}} \sqrt{x^2 + y^2 + z^2}\, dzdydx$. (10)

(8) Find the work down by the force field $\overrightarrow{F}(x, y) = y^3\overrightarrow{i} - x^3\overrightarrow{j}$ on a particle that moves along the boundary of the region $R = \{(x, y) : 4 \le x^2 + y^2 \le 16\}$, and $x \ge 0$. (10)

(9) Use the Divergence theorem to evaluate $\int\int_\sigma \overrightarrow{F} \cdot \overrightarrow{n} ds$, where $\overrightarrow{n}$ is the outer unit normal vector to σ,
$\overrightarrow{F}(x, y, z) = (6xy^2)\overrightarrow{i} - (x \sin z)\overrightarrow{j} + (2z^3)\overrightarrow{k}$, and σ is the surface of the solid bounded by the cylinder $y^2 + z^2 = 4$ and the planes $x = -2$ and $x = 2$. (10)

(10) Use Stokes' theorem to find the work down by the force field
$\overrightarrow{F}(x, y, z) = (2y - e^x)\overrightarrow{i} + (z + \sin y)\overrightarrow{j} + (x)\overrightarrow{k}$, in moving an object along an arch of the curve $\overrightarrow{r}(t) = (\cos t)\overrightarrow{i} - (\sin t)\overrightarrow{j} + (\sin 2t)\overrightarrow{k}, 0 \le t \le 2\pi$. (10)