Math 271 **Final (Version 3)**

Name........................J Number..................Score..................

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(1) Find an equation of the line through $(1,-1,2)$ and intersects the line $\begin{cases} x = t-3 \\ y = t-6 \\ z = -t+8 \end{cases}$ at a right angle. (10)

(2) Find the limits. (10)

(a). $\lim_{(x,y)\to(0,0)} \frac{3xy}{\sqrt{x^2+4y^2}}$;

(b). $\lim_{(x,y)\to(0,0)} \frac{3x^2y}{x^4+2y^2}$.

(3) Find the partial derivatives of z with respect to x, and y respectively: (10)

(a). $z = e^s \sin t$, where $s = xy, t = \sqrt{x^2+y^2}$;

(b). $\ln(y+xz) = 2 - xy^2z^2$.

(4) The temperature T at a point (x,y) on a semi-circular plate is given by $T(x,y) = 3x^2y - y^3 + 273$. (10)

(a). Find the temperature at $(1,2)$;

(b). Find the rate of change of temperature at $(1,2)$ in the direction of $\overrightarrow{a} = \overrightarrow{i} - 2\overrightarrow{j}$;

(c). Find a unit vector in the direction in which the temperature increases most rapidly at $(1,2)$ and find this maximum rate of change.

(5) Consider the surface $z = x^3 - 9xy + y^3$.(10)

(a). Find all the critical points on this surface;

(b). At each of the critical points determine if a relative maximum, relative minimum or saddle point occurs.

(6) Consider the double integral $\int_0^\pi \int_y^\pi \frac{\sin x}{x} dxdy$. (10)

(a). Sketch the region of integration;

(b). Evaluate the integral.

(7) Evaluate the integral by changing it to the cylindrical coordinates: $\int_0^3 \int_0^{\sqrt{9-x^2}} \int_{\sqrt{x^2+y^2}}^3 \sqrt{x^2+y^2}\, dzdydx$. (10)

(8) Find the work down by the force field $\overrightarrow{F}(x,y) = y^3\overrightarrow{i} - x^3\overrightarrow{j}$ on a particle that moves along the boundary of the region $R = \{(x,y) : 1 \le x^2+y^2 \le 9\}$, and $x \ge 0$. (10)

(9) Use the Divergence theorem to find the flux of $\overrightarrow{F}$ across σ: $\int\int_\sigma \overrightarrow{F} \cdot \overrightarrow{n}\, ds$, where $\overrightarrow{n}$ is the outer unit normal vector to σ, $\overrightarrow{F}(x,y,z) = (xy^2)\overrightarrow{i} + (x\sin z)\overrightarrow{j} + (x^2z)\overrightarrow{k}$, and σ is the surface of the solid bounded above by $z = 4 - x^2 - y^2$ and below by $z = 0$. (10)

(10) Use Stokes' theorem to find the work down by the force field $\overrightarrow{F}(x,y,z) = (2y - \ln x)\overrightarrow{i} + (z + \sin y)\overrightarrow{j} + x\overrightarrow{k}$, in moving an object along an arch of the curve $\overrightarrow{r}(t) = (\cos t)\overrightarrow{i} + (\sin t)\overrightarrow{j} + (\sin 2t)\overrightarrow{k}, 0 \le t \le 2\pi$. (10)