Math 271 **Final (Version 2)**

Name........................J Number..................Score..................

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(1) Find parametric equation of the line through $(-1, 0, 2)$ and parallel to the line $\begin{cases} 2x - y = -3 \\ x + 2y - 2z = 8. \end{cases}$ (10)

(2) Find the unit tangent vector $\overrightarrow{T}$ and the unit normal vector $\overrightarrow{N}$ for $\overrightarrow{r}(t) = (2\sin t)\overrightarrow{i} + (2\cos t)\overrightarrow{j} + t\overrightarrow{k}$ at $t = 0$. (10)

(3) Let $f(x, y, z) = \frac{x}{y+z}$, (10)
  (a). find $df(x, y, z)$;
  (b). find $\nabla f(x, y, z)$ at $(4, 1, 1)$;
  (c). find $D_{\overrightarrow{a}} f$ at p, where $p = (4, 1, 1), and \overrightarrow{a} = -3\overrightarrow{i} + 2\overrightarrow{j} + 6\overrightarrow{k}$;

(4) Use Lagrange multipliers to find the Maximum and minimum values of $f(x, y, z) = xy + 2yz + 2xz$ subject to constraint $xyz = 32$.(10)

(5) Evaluate the integral by changing it to the polar coordinates: $\int\int_D \frac{dxdy}{(x^2+y2)^2}$, where $D = \{(x, y) : x \leq x^2 + y^2 \leq 1\}$. (10)

(6) Without changing the integrand express the integral as an equivalent one in which the order of integral is $dzdxdy$: $\int_0^3 \int_0^{1-\frac{z}{3}} \int_0^{2-2x-\frac{2z}{3}} f(x, y, z)dydxdz$. (10)

(7) Evaluate the integral $\int_C (y^2 - \tan^{-1} x)dx - (2x + \cos y)dy$, where C is the boundary of the region formed by the parabola $y = x^2$, x-axis and the line $x = 1$ with a counterclockwise orientation. (10)

(8) Consider the following coordinate systems in $\mathbb{R}^2$: $\begin{cases} u = 2x + y \\ v = x - 2y \end{cases}$, (10)
  (a). draw the pictures of the lines $u = 0, u = 1, v = 0$, and $v = 1$;
  (b). compute $\frac{\partial(x,y)}{\partial(u,v)}$;
  (c). find the area of the figure bounded by above lines using u-v coordinates;

(9) Use the Divergence theorem to find the flux of $\overrightarrow{F}(x, y, z) = (3xy^2)\overrightarrow{i} - (x^3 e^z)\overrightarrow{j} + (3x^2 z)\overrightarrow{k}$ across the surface of the solid bounded above by $z = 4$ and below by $z = x^2 + y^2$. (10)

(10) Use Stokes' theorem to estimate the integral $\int_C \overrightarrow{F} \cdot d\overrightarrow{r}$, where $\overrightarrow{F}(x, y, z) = (-2x^2 z)\overrightarrow{i} - (2xy^2)\overrightarrow{j} + (8z)\overrightarrow{k}$, C is the curve of intersection of the plane $x + y - z = 1$ and the circular cylinder $x^2 + y^2 = 25$ oriented counterclockwise as viewed from above. (10)