Math 271 **Final (Version 1)**

Name.....J Number.....Score.....

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(1) Find an equation of the line through $(1,-1,2)$ and intersects the line
$\begin{cases} x = t-3 \\ y = t-6 \\ z = -t+8 \end{cases}$ at a right angle. (10)

(2) Find the unit tangent vector $\overrightarrow{T}$ and the unit normal vector $\overrightarrow{N}$ for $\overrightarrow{r}(t) = (e^t \sin t)\overrightarrow{i} - (e^t \cos t)\overrightarrow{j} + e^t \overrightarrow{k}$ at $t = 0$. (10)

(3) Solve the partial differential equation: $2\frac{\partial f}{\partial x} - \frac{\partial f}{\partial y} = 0$, using the indicated change of variables: $u = x + y, v = x + 2y$. (10)

(4) Let $f(x,y) = \tan^{-1}\frac{y}{x}$, (10)
   (a). find $df(x,y)$;
   (b). find $\nabla f(x,y)$ at $(-1,2)$;
   (c). find $D_{\overrightarrow{a}} f$ at p, where $p = (1,-1)$, and $\overrightarrow{a} = 3\overrightarrow{i} + 4\overrightarrow{j}$;

(5) Use Lagrange multipliers to find the Maximum and minimum values of $f(x,y,z) = 2xy + 2yz + xz$ subject to constraint $xyz = 4$.(10)

(6) Evaluate the integral by changing it to the polar coordinates: $\int\int_D \frac{dxdy}{(x^2+y2)^2}$, where $D = \{(x,y) : y \le x^2 + y^2 \le 1\}$. (10)

(7) Without changing the integrand express the integral as an equivalent one in which the order of integral is reversed (i.e. $dxdydz$): $\int_0^1 \int_0^{2-2x} \int_0^{3-3x-\frac{3y}{2}} f(x,y,z)dzdydx$. (10)

(8) Evaluate the integral $\int_C y^3dx - x^3dy$, where C is the boundary of the region $R = \{(x,y) : 1 \le x^2 + y^2 \le 9$, and $x \ge 0\}$ with a counterclockwise orientation. (10)

(9) Use the Divergence theorem to evaluate $\int\int_\sigma \overrightarrow{F} \cdot \overrightarrow{n}ds$, where $\overrightarrow{n}$ is the outer unit normal vector to σ, $\overrightarrow{F}(x,y,z) = (x^2y)\overrightarrow{i} + (xz)\overrightarrow{j} + (xyz)\overrightarrow{k}$, and σ is the surface of the solid bounded above by $z = 9$ and below by $z = x^2 + y^2$. (10)

(10) Use Stokes' theorem to estimate the integral $\int_C \overrightarrow{F} \cdot d\overrightarrow{r}$, where $\overrightarrow{F}(x,y,z) = (2y)\overrightarrow{i} + z\overrightarrow{j} + (4x)\overrightarrow{k}$, C is the triangle with vertices $(3,0,0), (0,3,3), (0,0,0)$ with a counterclockwise orientation looking down the positive z-axis. (10)