

Name.....SSN.....Score.....

## Show all work

- Find the limits (16)
  - $\lim_{x \rightarrow -\infty} \frac{\sqrt{4x^2-3}}{x}$ ;
  - $\lim_{x \rightarrow -1} \frac{x^2-x-2}{x^2-1}$ ;
  - $\lim_{x \rightarrow 0^+} (\sin x)^{\sin x}$ ;
  - $\lim_{x \rightarrow 2} \frac{\sin(x-2)}{x^2-4}$ .
- Find the derivatives of  $y = f(x)$  with respect to  $x$  (16)
  - $f(x) = \cos(\sin(x+1))$ ;
  - $f(x) = \frac{x}{\sqrt{1+x^2}}$ ;
  - $f(x) = \sqrt{x} \sin \sqrt{x}$ ;
  - $5y^2 + \sin y = x^2$ .
- Find the integrals (20)
  - $\int \frac{x}{(3x^2+1)^2} dx$ ;
  - $\int \tan^2 x \sec^2 x dx$ ;
  - $\int_0^1 (x^3 - 2x + 1) dx$ ;
  - $\int_{-1}^0 6t^2(t^3 + 1)^{10} dx$ .
- Use a linear approximation to estimate the following values (10)
  - $\sqrt[3]{63}$ ;
  - $\sin 31^\circ$ .
- If  $f(x) = \sqrt{x+4}$ , and  $(f \circ g)(x) = |x-1|$ , find  $g(x)$ . (5)
- If a ball is thrown vertically upward with a velocity of 80 ft/s, then its height in feet after  $t$  seconds is  $s = 80t - 16t^2$ . What is the velocity of the ball when it is 96 ft above the ground on its way up? On its way down? (5)
- An open box is to be made from a 6ft by 8ft rectangular piece of sheet metal by cutting out squares of equal size from the four corners and bending up the sides. Find the maximum volume that the box can have. (10)
- Sketch the graph of  $y = x^3 - 3x + 2$ . Find x- and y- intercepts; plot the stationary points and the inflection points and determine the intervals where  $y$  is increasing and decreasing, where the graph is concave up and concave down. (10)
- Use the second part of fundamental theorem of calculus: (8)
  - Find the derivative  $\frac{d}{dx} \int_{x^2}^{\sin x} \sin t^2 dt$ ;
  - Prove that the function  $F(x) = \int_{2x}^{4x} \frac{2}{t} dt$  is a constant on the interval  $(-\infty, 0)$ .