171 Calculus 1 Review 2

In the following problems find the derivative. Simplify if possible.

1. $f(x) = 3^{2x}\csc(5x)$.

We combine the product and the chain rules and use the formulas

$\frac{d(a^x)}{dx} = a^x \ln a$ and $\frac{d(\csc x)}{dx} = -\csc x \cot x$. Thus we obtain

$$\frac{df}{dx} = 3^{2x} \times \ln 3 \times 2\csc(5x) - 3^{2x}\csc(5x)\cot(5x)\times 5 = 3^{2x}\csc(5x)[2\ln 3 - 5\cot(5x)].$$

2. $f(x) = \frac{x^4 - x^2 + 1}{x^4 + x^2 + 1}$. We use the quotient rule $\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$.

$$f'(x) = \frac{(4x^3 - 2x)(x^4 + x^2 + 1) - (x^4 - x^2 + 1)(4x^3 + 2x)}{(x^4 + x^2 + 1)^2} =$$

$$= \frac{4x^7 + 4x^5 + 4x^3 - 2x^5 - 2x^3 - 2x - 4x^7 + 4x^5 - 4x^3 - 2x^5 + 2x^3 - 2x}{(x^4 + x^2 + 1)^2} =$$

$$= \frac{4x^5 - 4x}{(x^4 + x^2 + 1)^2} = \frac{4x(x^4 - 1)}{(x^4 + x^2 + 1)^2}.$$

3. $f(x) = \sqrt{\cot(e^{\sin x})}$. The problem is on the application of the chain rule.

$$\frac{df}{dx} = \frac{1}{2\sqrt{\cot(e^{\sin x})}}(-\csc^2(e^{\sin x}))e^{\sin x}\cos x.$$

4. $f(x) = \frac{\ln x + \sin x}{\ln x + \cos x}$. Like in Problem 2 we apply the quotient rule.

$$\frac{df}{dx} = \frac{(1/x + \cos x)(\ln x + \cos x) - (\ln x + \sin x)(1/x - \sin x)}{(\ln x + \cos x)^2} =$$

$$= \frac{(1/x)\ln x + (1/x)\cos x + \cos x \ln x + \cos^2 x - (1/x)\ln x + \sin x \ln x - \sin x(1/x) + \sin^2 x}{(\ln x + \cos x)^2} =$$

$$= \frac{(1/x)(\cos x - \sin x) + \ln x(\cos x + \sin x) + 1}{(\ln x + \cos x)^2}$$

5. Use implicit differentiation to find the slope-intercept equation of the tangent line to the curve $x^3 + y^4 = y^3 + x^4$ at the point (1, 1).

After differentiating both parts of the relation we get

$3x^2 + 4y^3 \dfrac{dy}{dx} = 3y^2 \dfrac{dy}{dx} + 4x^3$. Plugging in $x = 1$ and $y = 1$ we obtain

$3 + 4\dfrac{dy}{dx}(1,1) = 3\dfrac{dy}{dx}(1,1) + 4$ whence $\dfrac{dy}{dx}(1,1) = 1$. An equation of the tangent line at point (1, 1) in the point-slope form is $y - 1 = x - 1$ whence the slope-intercept form is $y = x$.

6. $f(x) = (\sec x)^{\csc x}$. We use the formula for logarithmic differentiation.

$\dfrac{df}{dx} = f(x)\dfrac{d(\ln(f(x)))}{dx}$. We have $\ln(f(x)) = \csc x \ln(\sec x)$ whence

$\dfrac{d(\ln(f(x)))}{dx} = -\csc x \cot x \ln(\sec x) + \csc x(1/\sec x)\sec x \tan x$

$= -\csc x \cot x \ln(\sec x) + \sec x$

(We have used the formula $\csc x \tan x = \sec x$).

Finally, $\dfrac{df}{dx} = (\sec x)^{\csc x}(\sec x - \csc x \cot x \ln(\sec x))$.