The following is copied from Wikipedia.

Circumference

The circumference *C* of an ellipse is $4aE(\varepsilon)$, where the function *E* is the complete elliptic integral of the second kind. The exact infinite series is:

$$C = 2\pi a \left[1 - \left(\frac{1}{2}\right)^2 \varepsilon^2 - \left(\frac{1\cdot 3}{2\cdot 4}\right)^2 \frac{\varepsilon^4}{3} - \left(\frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\right)^2 \frac{\varepsilon^6}{5} - \dots\right]$$

or

$$C = 2\pi a \sum_{n=0}^{\infty} \left\{ - \left[\prod_{m=1}^{n} \left(\frac{2m-1}{2m}\right)\right]^2 \frac{\varepsilon^{2n}}{2n-1} \right\};$$

A good approximation is Ramanujan's:

$$C \approx \pi \left[3(a+b) - \sqrt{(3a+b)(a+3b)}\right]$$

or better approximation:

$$C \approx \pi (a+b) \left(1 + \frac{3\left(\frac{a-b}{a+b}\right)^2}{10 + \sqrt{4 - 3\left(\frac{a-b}{a+b}\right)^2}}\right);$$

For the special case where the minor axis is half the major axis, we can use:

$$C \approx \frac{\pi a(9 - \sqrt{35})}{2}$$

or the better approximation

$$C \approx \frac{a}{2}\sqrt{93 + \frac{1}{2}\sqrt{3}}$$

More generally, the arc length of a portion of the circumference, as a function of the angle subtended, is given by an incomplete elliptic integral. The inverse function, the angle subtended as a function of the arc length, is given by the elliptic functions.

[edit] Surface area

The surface area of an ellipsoid is given by:

$$2\pi\left(c^2+b\sqrt{a^2-c^2}E(\oe,m)+\frac{bc^2}{\sqrt{a^2-c^2}}F(\oe,m)\right),$$

where

$$\oe=\arccos\left(\frac{c}{a}\right)\ (\text{oblate})\ \text{ or }\ \arccos\left(\frac{a}{c}\right)\ (\text{prolate}),$$

is the modular angle, or **angular eccentricity**; $m=\frac{b^2-c^2}{b^2\sin(\oe)^2}$ and $E(\oe,m)$, $F(\oe,m)$ are the incomplete elliptic integrals of the first and second kind.

Unlike the surface area of a sphere, the surface area of a general ellipsoid cannot be expressed exactly by an elementary function.

An approximate formula is:

$$\approx 4\pi\left(\frac{a^pb^p+a^pc^p+b^pc^p}{3}\right)^{1/p}.$$

Where $p \approx 1.6075$ yields a relative error of at most 1.061% (Knud Thomsen's formula); a value of $p = 8/5 = 1.6$ is optimal for nearly spherical ellipsoids, with a relative error of at most 1.178% (David W. Cantrell's formula).

Exact formulae can be obtained for the case $a = b$ (i.e., a spherical equator):

If oblate: $$2\pi\left(a^2+c^2\frac{\operatorname{arctanh}(\sin(\oe))}{\sin(\oe)}\right);$$

If prolate: $$2\pi\left(a^2+c^2\frac{\oe}{\tan(\oe)}\right);$$

In the "flat" limit of $c \ll a, b$, the area is approximately $2\pi ab$.